Towards a sound massive graviy

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Outline

- 0. Massive gravity
- 1. Ghost problem
- 2. Static universe problem
- 3. Towards sound massive cosmology
- 4. Fitting
- 5. Discussions

Massive gravity

- Scalar field

$$\frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2$$

- Proca field

$$-\frac{1}{16\pi}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) + \frac{m^2c^2}{8\pi\hbar^2}A^\nu A_\nu$$

- Gravity field?
-

$$R + C g_{\mu\nu} g^{\mu\nu} \,?$$

A proto type of massive gravity

- Linear approximation of the de Sitter
- $\Box h_{\mu\nu} - 2\Lambda h_{\mu\nu} = 0$

$$m_g = \sqrt{2\Lambda} = \sqrt{2\Lambda}\hbar c^{-1}$$

- Liao Liu, Zheng Zhao, arXiv:gr-qc/0404040

Fierz-Pauli approach

$$S_{FP} = S_{EH}[h] - \frac{1}{4}M_p^2 m^2 \int d^4x \, [h^\mu_\nu \, h^\nu_\mu - a\,(h^\mu_\mu)^2\,]$$

dRGT model

- Key point of massive gravity: Potential of gravity
- ? $V=V(g_{ab})$
- $V=V(g_{ab}, f_{ab})$

Simlified dRGT

- potential

$$V = -2N\sqrt{\gamma}(\mathrm{tr}\sqrt{I^{\mu}{}_{\alpha}} - A), \qquad I^{\mu}{}_{\alpha} = g^{\mu\nu} f_{\nu\alpha}.$$

Freedoms in Maxwell

- EM field: Aa
- Cosnraint: $\vec{\nabla}\cdot\vec{E}=0\,,$ $\vec{\nabla}\cdot\vec{B}=0\,.$
- Freedoms: 2
- n-d: n-2

ADM decomposition

$$ds^2 = -(N^2 - N_i N^i)dt^2 + 2N_i dt dx^i + \gamma_{ij} dx^i dx^j,$$

$$g^{ab} = \frac{1}{N^2}\left[-\left(\frac{\partial}{\partial t}\right)^a \left(\frac{\partial}{\partial t}\right)^b + \left(\frac{\partial}{\partial t}\right)^a N^i \left(\frac{\partial}{\partial x^i}\right)^b + \left(\frac{\partial}{\partial t}\right)^b N^i \left(\frac{\partial}{\partial x^i}\right)^a + (N^2\gamma^{ij} - N^i N^j)\left(\frac{\partial}{\partial x^i}\right)^a \left(\frac{\partial}{\partial x^j}\right)^b\right]$$

Freedoms in GR

$$S = \frac{1}{2\kappa^2} \int d^4x (\pi^{ij} \dot{\gamma}_{ij} + N_\mu R^\mu)$$

$$R^0 = \sqrt{\gamma} \left[\mathbf{R} + \frac{1}{\gamma} (\frac{\pi^2}{2} - \pi_{ij} \pi^{ij}) \right],$$

$$R^i = 2\sqrt{\gamma}\, \nabla_j \left(\frac{\pi^{ij}}{\sqrt{\gamma}} \right),$$

- Four constraints

$$R^\mu = 0$$

$$^3R - K_{ab} K^{ab} + K^2 = 0\,,$$

$$\mathrm{D}_a K^a{}_c - \mathrm{D}_c K = 0\,.$$

Freedoms in GR

- Freedoms: 6-4=2
- n-d: n(n-1)/2-n=n(n-3)/2

Freedoms in massive gravity

$$S = \frac{1}{2\kappa^2}\int d^4x \left(\pi^{ij}\dot{\gamma}_{ij} + N_\mu R^\mu + m^2 V(N_\mu, \gamma_{ij}, f)\right),$$

$$V = -2N\sqrt{\gamma}(\mathrm{tr}\sqrt{I^\mu{}_\alpha} - A), \qquad I^\mu{}_\alpha = g^{\mu\nu} f_{\nu\alpha}.$$

$$R^\mu + m^2 \frac{\partial V}{\partial N_\mu} = 0.$$

- Two polirizations, three translational freedoms, one possible ghost

Condition of ghost-free

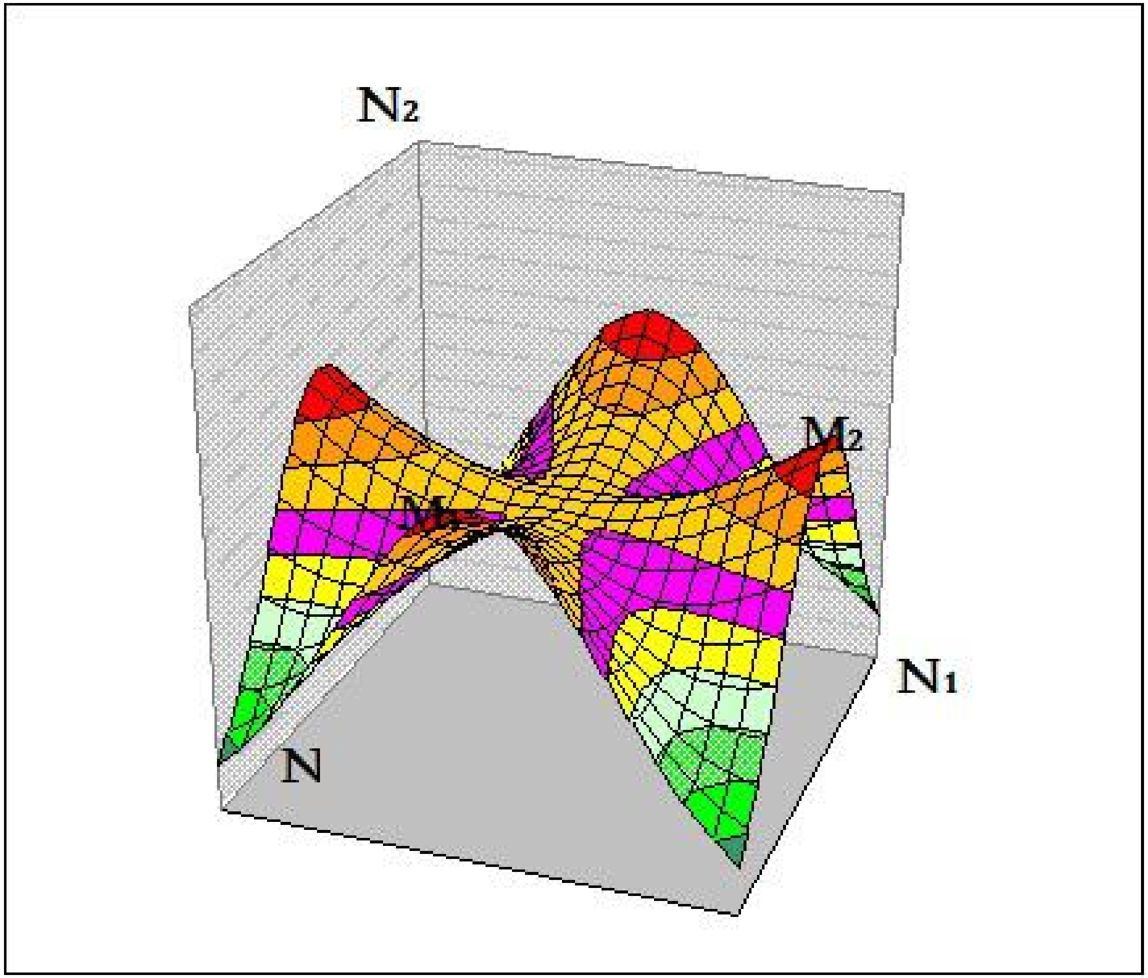
- Hamiltonian constraint suppresses the ghost.

$$F(R^{\mu}) = 0, \quad \longrightarrow \quad F(\frac{\partial V}{\partial N_{\mu}}) = 0.$$

$$\longrightarrow \quad C(N, N_i) = 0.$$

$$S = \frac{1}{2\kappa^2}\int d^4x N F(R^{\mu}) + \dots ,$$

ADM reparametrization



ADM reparametrization

$$M_i = M_i(N, N_j).$$

Inversely, we have

$$N_j = N_j(M_i, N).$$

ADM reparametrization

$$S = \frac{1}{2\kappa^2} \int d^4x N F(R^\mu) + ...$$

$$N^i = P^i(M^k, \gamma_{kj}) + N Q^i(M^k, \gamma_{kj}).$$

Squre-root of a matrix

-
- an $n \times n$ matrix has 2^n squre roots at least

For example the matrix diag(1,1) has four square roots:

diag(1,1), diag(1,-1), diag(-1,1), and diag(-1,-1).

Massive term

$$N^2 I^{\mu}{}_{\nu} = \begin{bmatrix} -f_{00} + N^i f_{i0} & -f_{0j} + N^i f_{ij} \\ N^2\gamma^{ij} f_{j0} + N^i f_{00} - N^i N^j f_{j0} & N^2\gamma^{ik} f_{kj} + N^i f_{0j} - N^i N^k f_{kj} \end{bmatrix}$$

Rank-1 f

$$f = \begin{pmatrix} f_{00} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$N(I^{\mu}{}_{\nu})^{\frac{1}{2}} = \sqrt{-f_{00}} \begin{bmatrix} 1 & \mathbf{0}_1 \\ -N^i & \mathbf{0}_2 \end{bmatrix},$$

$$V = -2\sqrt{\gamma}\sqrt{-f_{00}}.$$

- No transformation is required.

Rank-3 f

$$f_{\mu\nu} = \begin{bmatrix} 0 & \mathbf{0_1} \\ \mathbf{0_3} & f_{ij} \end{bmatrix}$$

$$c^i_j = M^i M^k f_{kj},$$

$$L^i{}_j = \mathrm{tr}(\mathrm{c^i{}_j})\delta^\mathrm{i}{}_\mathrm{j} + \mathrm{c^i{}_j}.$$

$$L^{-1} = \left(\mathrm{tr}(\mathrm{c^i{}_j})\mathsf{E} + \mathrm{MM^Tf}\right)^{-1},$$

$$D^i{}_j = \sqrt{\gamma^{ij} f_{jm} L^m_n}\ \left(L^{-1}\right)^n_j$$

The transformation

$$N^i = M^i + N D^i{}_k M^k$$

$$S = \frac{1}{2\kappa^2} \int d^4x \left[\pi^{ij}\dot{\gamma}_{ij} + N\left(R^0 - 2m^2\sqrt{\gamma(\mathrm{trc^i{}_j})}\ \mathrm{trD^i{}_j} - 2 \right) + M_i R^i - 2m^2\sqrt{\gamma(\mathrm{trc^i{}_j})} \right]$$

- Defromed H-constraint

$$R^0 - 2m^2\sqrt{\gamma(\mathrm{trc^i{}_j})}\ \mathrm{trD^i{}_j} - 2 = 0.$$

Deformed H-constraint in explicit form

$$R^i + m^2 \frac{\partial V}{\partial M_i} = 0.$$

$$M_i = M_i(R^j).$$

$$D^i{}_j = D^i{}_j(M_k).$$

$$R^0 - 2m^2\sqrt{\gamma(\mathrm{trc^i{}_j})}\ \mathrm{trD^i{}_j} - 2 = 0.$$

Rank-2 metric

$$f_{\mu\nu} = \begin{bmatrix} 0_{\mathbf{2}\times\mathbf{2}} & 0_{\mathbf{2}\times\mathbf{2}} \\ 0_{\mathbf{2}\times\mathbf{2}} & f_{mn} \end{bmatrix}$$

Can we get more constraints

- No

Why acceleration in massive cos

- A massless intermediate boson leads to a Newton-like potential 1/r.
- A massive intermediate boson leads to a Yukawa-like potential $e^{-\alpha r}/r$ which implies a weakened force at large distance.
- Some properties mimicking the prototype massive cosmology.

$$m \sim \alpha \sim H_0$$

Full dRGT

Structure of dRGT

$$V(g,f) = m^2 \sum^{4} c_i \mathcal{U}_i(g,f),$$

$$\mathcal{U}_1 = [\mathcal{K}],$$

$$\mathcal{U}_2 = [\mathcal{K}]^2 - [\mathcal{K}^2],$$

$$\mathcal{U}_3 = [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3],$$

$$\mathcal{U}_4 = [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4].$$

$$\mathcal{K}^\mu_\nu = \left(\sqrt{g_M^{-1} f_M}\right)^\mu_\nu.$$

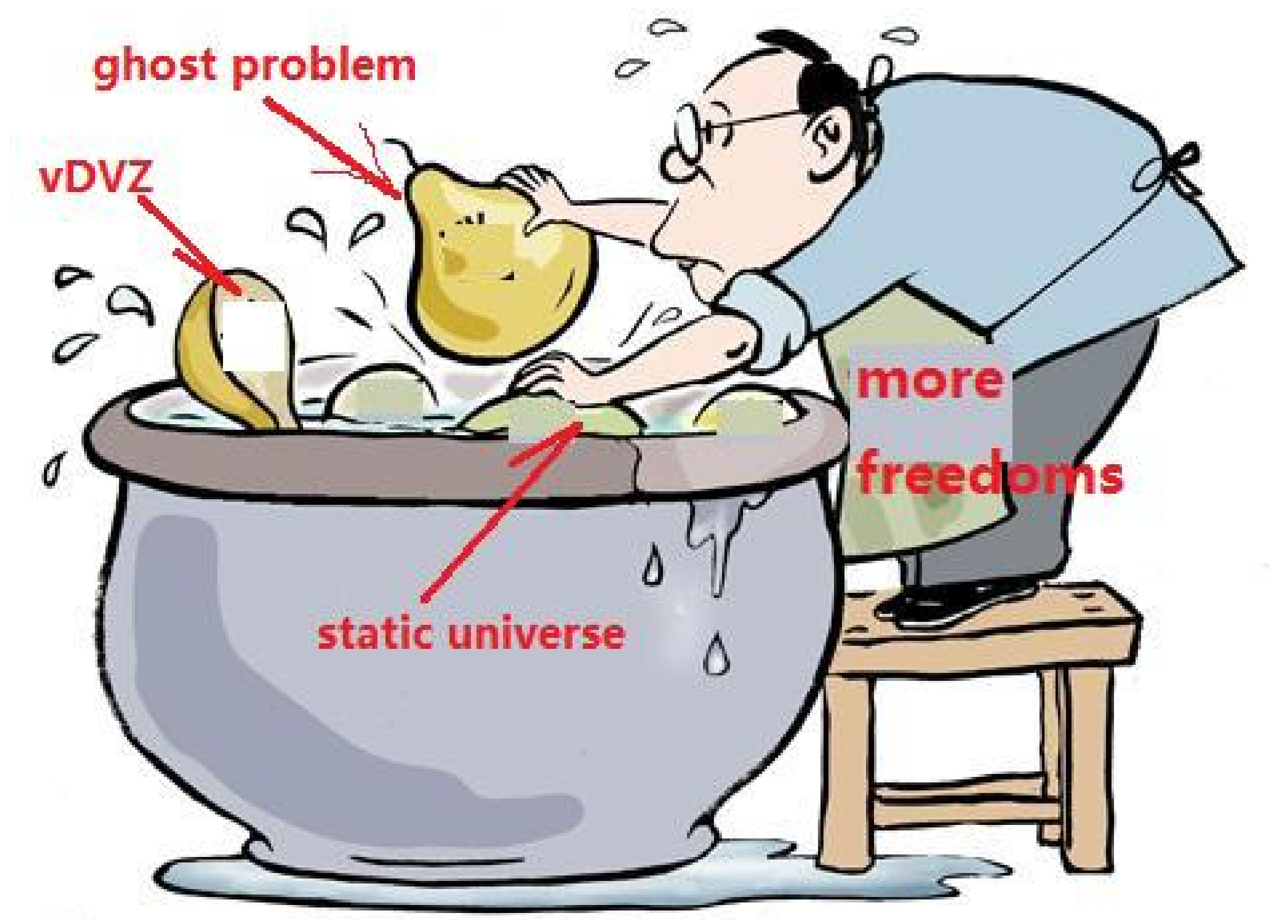
Static problem

$$\phi^a = \delta^a_\mu x^\mu$$

$$ds^2 = -N^2(t)dt^2 + a^2 d\vec{x}^2.$$

$$S = \int d^4x \left(A(a,\ \dot{a}) + Nm^2(c_1 a^3 + 6c_2 a^2 + 18c_3 a + 24c_4)\right).$$

When a gourd is pressed down，a gourd ladle is floated （按下葫蘆起來瓢）



More freedoms

- G. DAmico, G. Gabadadze, L. Hui and D. Pirtskhalava, Phys. Rev. D 87, 064037 (2013)doi:10.1103/PhysRevD.87.064037 [arXiv:1206.4253 [hep-th]]; A. E. Gumrukcuoglu, K. Koyama and S. Mukohyama, Phys. Rev. D 96, no. 4, 044041 (2017)doi:10.1103/PhysRevD.96.044041 [arXiv:1707.02004 [hep-th]]; A. De Felice and S. Mukohyama, Phys.Lett. B 728, 622 (2014) doi:10.1016/j.physletb.2013.12.041 [arXiv:1306.5502 [hep-th]]; G. Gabadadze,R. Kimura and D. Pirtskhalava, Phys. Rev. D 90, no. 2, 024029 (2014) doi:10.1103/PhysRevD.90.024029 [arXiv:1401.5403 [hep-th]];
- G. Gabadadze, K. Hinterbichler, J. Khoury, D. Pirtskhalava and M. Trodden, Phys. Rev. D86, 124004 (2012) doi:10.1103/PhysRevD.86.124004 [arXiv:1208.5773 [hep-th]]; M. Andrews, G.Goon, K. Hinterbichler, J. Stokes and M. Trodden, Phys. Rev. Lett. 111, no. 6, 061107 (2013)doi:10.1103/PhysRevLett.111.061107 [arXiv:1303.1177 [hep-th]].....
- S. Anselmi, S. Kumar, D. Lopez Nacir and G. D. Starkman, Phys. Rev. D 96, no. 8, 084001 (2017)doi:10.1103/PhysRevD.96.084001 [arXiv:1706.01872 [astro-ph.CO]].

Singular ref metric: to press the gourd and gourd ladle at the same time without introducing any more freedom

$$f_{\mu\nu} = \text{diag}(0, 1, 1, 1).$$

$$V = m^2 \left(\frac{3c_1}{a} + \frac{6c_2}{a^2} + \frac{6c_3}{a^3} \right)$$

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho - m^2 \left(\frac{c_1}{2a} + \frac{c_2}{a^2} + \frac{c_3}{a^3} \right),$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + m^2 \left(-\frac{c_1}{4a} + \frac{c_3}{2a^3} \right).$$

Density and pressure

$$\rho_g = -\frac{3m^2}{8\pi G}\left(\frac{c_1}{2a} + \frac{c_2}{a^2} + \frac{c_3}{a^3}\right),$$

$$p_g = \frac{m^2}{8\pi G}\left(\frac{c_1}{a} + \frac{c_2}{a^2}\right).$$

Superconductor

- diffeomorphism invariant in bulk
- no momentum dissipation

Normal conductor

- The effects of the lattice
- Momentum dissipation
- A natural model: massive gravity
- The ghost problem of massive gravity with a singular reference metric is solved in

Li-Ming Cao, Yuxuan Peng, Yun-Long Zhang, arXiv:1511.04967

HZ, Xin-Zhou Li, arXiv:1510.03204

Dynamical ref metric

$$f_{\mu\nu} = b^2(t)\mathrm{diag}(0,1,1,1)$$

$$\sqrt{-g}\, V = 3m^2\left(c_1 b a^2 + 2c_2 b^2 a + 2c_3 b^3\right)$$

E-L equation for b

$$c_1 a^2 + 4c_2 b a + 6c_3 b^2 = 0.$$

$$b = \frac{-2c_2 \pm \sqrt{4c_2^2 - 6c_1 c_3}}{6c_3}\, a$$

- Define

$$B_{\pm} = \frac{-2c_2 \pm \sqrt{4c_2^2 - 6c_1c_3}}{6c_3}.$$

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho - m^2\left(\frac{c_1B_{\pm}}{2} + c_2B_{\pm}^2 + c_3B_{\pm}^3\right),$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + m^2\left(-\frac{c_1B_{\pm}}{4} + \frac{c_3B_{\pm}^3}{2}\right).$$

$$\rho_g = -\frac{3m^2}{8\pi G}\left(\frac{c_1B_{\pm}}{2} + c_2B_{\pm}^2 + c_3B_{\pm}^3\right),$$

$$p_g = \frac{m^2}{8\pi G}\left(c_1B_{\pm} + c_2B_{\pm}^2\right).$$

$$\dot{\rho}_g + 3H(\rho_g + p_g) \neq 0,$$

Semi-dynamical ref metric

$$f_{\mu\nu} = \mathrm{diag}(0, 1, b^2, b^2)$$

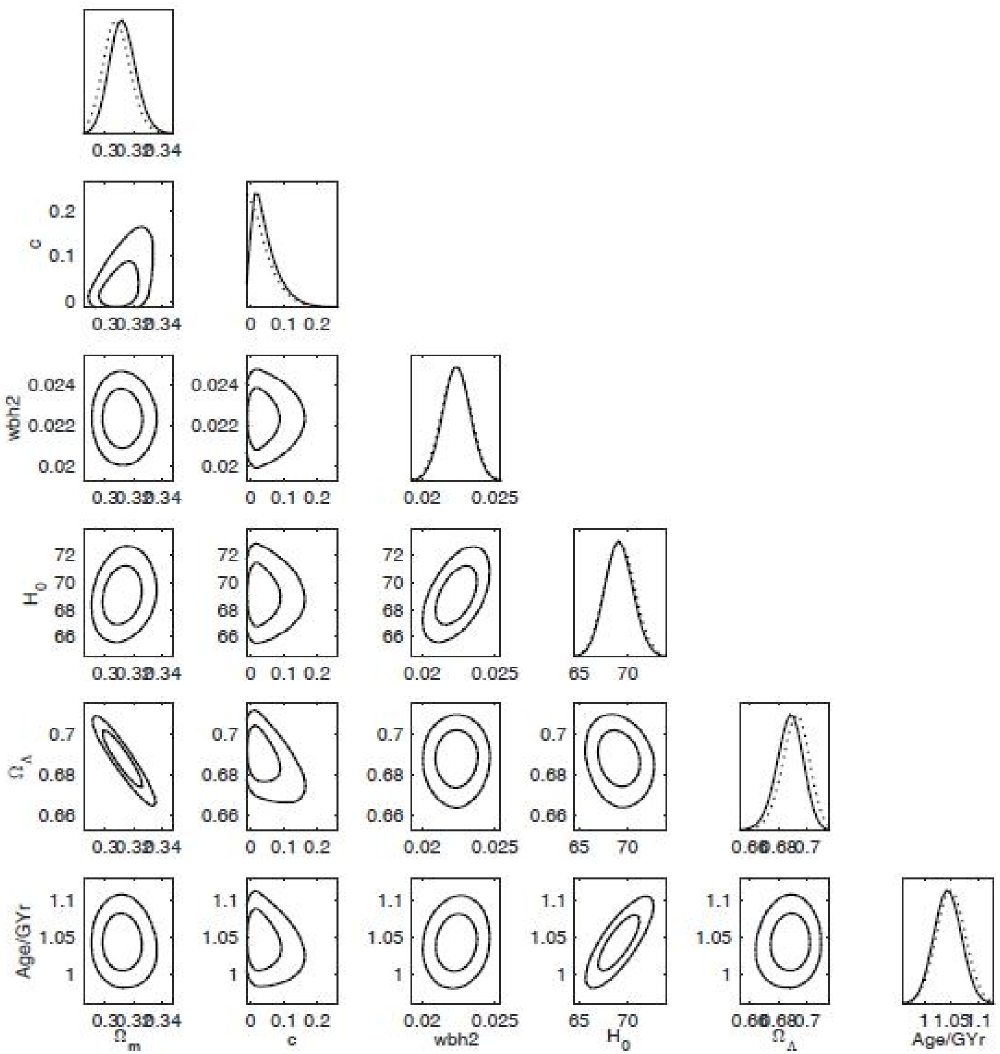
$$\sqrt{-g}\, V = 3m^2 \left(c_1 a^2(1+2b) + 2c_2 a(2b+b^2) + 6c_3 b^2\right)$$

$$b = -\frac{c_1 a^2 + 2c_2 a}{2c_2 a + 6c_3}$$

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho - m^2 \left(\frac{c_1(1+2b)}{6a} + \frac{c_2(2b+b^2)}{3a^2} + \frac{c_3 b^2}{a^3}\right),$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + m^2 \left(\frac{c_1(1-4b)}{12a} + \frac{c_2(b-b^2)}{3a^2} + \frac{c_3 b^2}{2a^3}\right).$$

Fitting



Discussions

Which ref metric is prefered, in principle?

More cosmological effects for massive gravity armed with singular ref metrics.

- *Thank you for your attention.*